

A new neutrino mass sum rule from inverse seesaw

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(Dated: February 24, 2013)

A class of discrete flavor-symmetry-based models predicts constrained neutrino mass matrix schemes that lead to specific neutrino mass sum-rules (MSR). One of these implies in a lower bound on the effective neutrinoless double beta mass parameter, even for normal hierarchy neutrinos. Here we propose a new model based on the S_4 flavor symmetry that leads to the new neutrino mass sum-rule and discuss how to generate a nonzero value for the reactor angle θ_{13} indicated by recent experiments, and the resulting correlation with the solar angle θ_{12} .

PACS numbers: 11.30.Hv, 12.15.Ff, 14.60.Pq

I. INTRODUCTION

The discovery of neutrino oscillations have provided a strong evidence of the non-vanishing neutrino masses, although their nature (if they are Dirac or Majorana particles) has so far remained elusive. The observation of neutrinoless double beta decay ($0\nu\beta\beta$) would provide an irrefutable confirmation of the Majorana nature of neutrinos [1]. Majorana neutrinos are characterized by a symmetric mass matrix whose parameters are restricted by the experimental data: the neutrino oscillation parameters, such as mixing angles and neutrino squared mass differences [2, 3], as well as the limits on the $0\nu\beta\beta$ effective mass parameter [4, 5]. Indeed, upcoming $0\nu\beta\beta$ experiments are expected to improve the sensitivity by up to about one order of magnitude [6–9].

The general neutrino mixing matrix containing the three mixing angles and the CP violating phases can be parametrized in different equivalent ways [10–12]. A particular *ansatz* of the mixing matrix is the Tribimaximal Mixing Matrix (TBM) [13] which, despite the fact of the non-zero value of the θ_{13} angle indicated by recent experiments [14, 15], can still be used as a good first approximation. Specially so taking into account that it can receive corrections from charged lepton diagonalization and/or from renormalization effects, depending on its scale of validity.

Several flavor models based in non-Abelian discrete symmetries predict a two-parameter neutrino mass matrix which imply a particular mixing matrix form, as is pointed out in [13]. In Ref. [16] it was noted that in these models

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only the following mass relations can be obtained,

$$\chi m_2^\nu + \xi m_3^\nu = m_1^\nu, \quad (1)$$

$$\frac{\chi}{m_2^\nu} + \frac{\xi}{m_3^\nu} = \frac{1}{m_1^\nu}, \quad (2)$$

$$\chi \sqrt{m_2^\nu} + \xi \sqrt{m_3^\nu} = \sqrt{m_1^\nu}, \quad (3)$$

$$\frac{\chi}{\sqrt{m_2^\nu}} + \frac{\xi}{\sqrt{m_3^\nu}} = \frac{1}{\sqrt{m_1^\nu}}, \quad (4)$$

where χ and ξ are free parameters that characterize each specific model. A classification of all models predicting TBM mixing which generate mass relations similar to the first three are given there. The last case is completely new and here we will present a model from first principles, implementing the inverse seesaw mechanism [17, 18] as well as a non-Abelian flavor symmetry [19], along the lines of Ref. [20], but adopting S_4 , instead of A_4 .

The inverse seesaw scheme constitutes the first example of a low-scale seesaw scheme [21] with naturally light neutrinos. The particle content is the same as that of the Standard Model (SM) except for the addition of a pair of two component gauge singlet leptons, ν_i and S_i , within each of the three generations, labeled by i . The isodoublet neutrinos ν_i and the fermion singlets S_i have the same lepton number, opposite with respect to that of the three singlets ν_i^c associated to the “right-handed” neutrinos. In the ν, ν^c, S basis the 9×9 neutral lepton mass matrix M_ν has the form:

$$M_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix}, \quad (5)$$

where Y_ν and M are arbitrary 3×3 complex matrices, while μ is symmetric due to the Pauli principle. Following the diagonalization seesaw method in [22] one obtains the effective light neutrino mass matrix as:

$$m_\nu \sim m_D^T M^{T-1} \mu M^{-1} m_D, \quad (6)$$

with the entry μ being very small. The diagram illustrating the mass generation through the inverse seesaw mechanism is shown in Figure I.

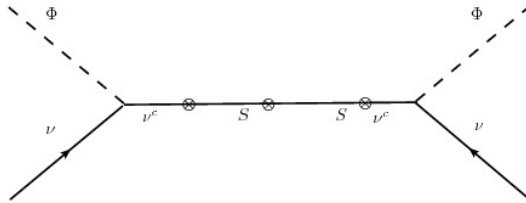


Figure 1: Inverse seesaw mechanism.

It is straightforward to show that if m_D and μ are both proportional to the identity, and

$$M \sim M_{TBM} = \begin{pmatrix} x & y & y \\ y & x+z & y-z \\ y & y-z & x+z \end{pmatrix}, \quad (7)$$

in the basis where the charged lepton mass matrix is diagonal. Here there is a specific (complex) relation among the parameters x, y and z [23], leaving only two free complex parameters, we obtain the sum mass rule in Eq. (4).

In the next section we give our model, in section III we present the predictions regarding the lower bound on the neutrinoless double beta decay amplitude, and discuss possible departures from tribimaximality, including a finite θ_{13} value. In the appendix C we present details on the symmetry structure, Yukawa couplings as well as the scalar potential of the model.

II. THE MODEL

Here we follow table I given in reference [20], where some possible schemes realizing the tri-bimaximal mixing pattern are summarized for the inverse seesaw case. From these we will implement case 1), since the other two cases, 2) and 3), correspond to the mass sum rule relations C) and A) respectively, which have already model realizations in the existing literature,

$$M_D \propto \mathcal{I} \quad \mu \propto \mathcal{I}, \quad M \propto \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & B \end{pmatrix}, \quad (8)$$

In contrast to Ref. [20] here we adopt the S_4 flavour symmetry, instead of A_4 .

In order to obtain the S_4 -based inverse seesaw model we assign the charge matter fields as in table I. Three right handed neutrinos ν_R are introduced, and three $SU(2)$ fermion singlets $S_i, i = 1, 2, 3$, the latter transforming as the $\mathbf{3}_1$ (note that ν^c and ν_R are conjugates, hence have opposite lepton number). All fermion fields in table I transform as the triplet $\mathbf{3}_1$ and the Higgs doublet as the trivial singlet $\mathbf{1}_1$.

On the other hand, in order to generate the desired mass matrix structures we introduce five flavon fields, $\phi_\nu, \phi'_\nu, \phi_l, \phi'_l, \phi''_l$ supplemented by the extra symmetries Z_3 and Z_2 , whose assignments are given in table II. The presence of these extra Abelian symmetries in the theory ensures the presence of adequate zeros in the neutrino mass matrix. We keep renormalizability of the Lagrangian by adding a Frogatt-Nielsen fermion χ and its conjugate χ^c , both singlets under the weak $SU(2)$ gauge group [24–27]. In table II we present the relevant quantum numbers of the matter fields in the theory under these extra symmetries.

	\bar{L}	ν_R	l_R	h	S	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
$SU(2)$	2	1	1	2	1	1	1	1	1	1	1	1	1
S_4	3_1	3_1	3_1	1_1	3_1	3_1	1_1	3_1	3_2	1_1	1_1	3_1	3_1
$U_l(1)$	-1	1	1	0	-1	0	0	0	0	0	2	1	-1

Table I: Fields and transformation properties under $SU(2)$, the S_4 flavor symmetry, and global lepton number $U_l(1)$

	\bar{L}	ν_R	l_R	h	S	ϕ_ν	ϕ'_ν	ϕ_l	ϕ'_l	ϕ''_l	σ	χ	χ^c
Z_3	ω^2	ω	1	1	1	ω^2	ω^2	ω	ω	ω	1	ω	ω^2
Z_2	+	+	+	+	-	-	-	+	+	+	+	+	+

Table II: Fields and their transformation properties under the Z_3 , and Z_2 flavor symmetries

The renormalizable Lagrangian relevant for neutrinos is

$$\mathcal{L}_\nu = Y_{D_{ij}} \bar{L}_i \nu_{R_j} h + Y_{\nu_{ij}}^k \nu_{R_i} S_j \phi_{\nu_k} + Y'_{\nu_{ij}} \nu_{R_i} S_j \phi'_\nu + \mu_{ij} S_i S_j \sigma, \quad (9)$$

while the renormalizable Yukawa terms involving the messenger fields is

$$\mathcal{L}_\chi = M_\chi \chi \chi^c + \bar{L} h \chi + \chi^c l_R \Phi_l + \chi^c l_R \Phi'_l + \chi^c l_R \Phi''_l. \quad (10)$$

After integrating out the messenger fields χ , the effective Lagrangian for charged leptons takes the form

$$\mathcal{L}_l = \frac{y_l}{\Lambda} (\bar{L} l_R) h \phi_l + \frac{y'_l}{\Lambda} (\bar{L} l_R) h \phi'_l + \frac{y''_l}{\Lambda} (\bar{L} l_R) h \phi''_l, \quad (11)$$

where Λ is the effective scale. This effective Lagrangian is responsible for charged lepton mass generation, as shown in figure 2.

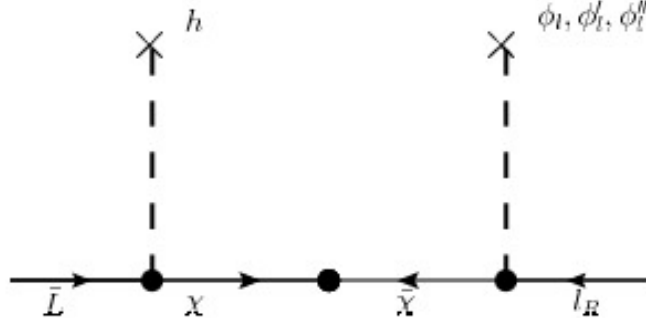


Figure 2: Diagram illustrating charged lepton mass generation.

In order to obtain the desired neutrino mixing matrix we require the flavon fields to have the following alignments:

$$\langle \phi_\nu \rangle = v_\nu (1, 0, 0), \quad \langle \phi_l \rangle = v_l (1, 1, 1), \quad \langle \phi_{l'} \rangle = v_{l'} (1, 1, 1), \quad (12)$$

where we also define $\langle \phi_{\nu'} \rangle = v_{\nu'}$, $\langle \phi_{l''} \rangle = v_{l''}$, $\langle \sigma \rangle = v_\sigma$ and $\langle h \rangle = v$. In appendix C we report the form of the potential (C2). We have verified that there exists a large portion of the parameter space where the required alignment is found to be a solution of minimization of the potential.

With these alignments the three 3×3 blocks in Eq. (5) and charged lepton matrices take the form

$$(\mu) = \begin{pmatrix} \mu v_\sigma & 0 & 0 \\ 0 & \mu v_\sigma & 0 \\ 0 & 0 & \mu v_\sigma \end{pmatrix}, \quad M_D = \begin{pmatrix} Y_D v & 0 & 0 \\ 0 & Y_D v & 0 \\ 0 & 0 & Y_D v \end{pmatrix}, \quad M = \begin{pmatrix} Y_{\nu'} v_{\nu'} & 0 & 0 \\ 0 & Y_{\nu'} v_{\nu'} & Y_\nu v_\nu \\ 0 & Y_\nu v_\nu & Y_{\nu'} v_{\nu'} \end{pmatrix}, \quad (13)$$

and

$$M_l = \begin{pmatrix} y_{l''} v_{l''} & y_l v_l - y_{l'} v_{l'} & y_l v_l + y_{l'} v_{l'} \\ y_l v_l + y_{l'} v_{l'} & y_{l''} v_{l''} & y_l v_l - y_{l'} v_{l'} \\ y_l v_l - y_{l'} v_{l'} & y_l v_l + y_{l'} v_{l'} & y_{l''} v_{l''} \end{pmatrix} \frac{v}{\Lambda}. \quad (14)$$

The charged lepton mass matrix, Eq. (14), is diagonalized by the "magic" matrix

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \quad (15)$$

On the other hand, by using Eqs. (5) and (6) it is straightforward to obtain the light neutrino mass matrix which takes the form

$$M_\nu = \begin{pmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{a^2+b^2}{(b^2-a^2)^2} & -\frac{2ab}{(b^2-a^2)^2} \\ 0 & -\frac{2ab}{(b^2-a^2)^2} & \frac{a^2+b^2}{(b^2-a^2)^2} \end{pmatrix} \quad (16)$$

where $a = Y_{\nu'} v_{\nu'} / (\sqrt{\mu v_\sigma} Y_D v)$ and $b = Y_\nu v_\nu / (\sqrt{\mu v_\sigma} Y_D v)$. In the basis where charged lepton mass matrix is diagonal, the light neutrino mass matrix is diagonalized by the TBM-form, and the corresponding eigenvalues are given by

$$\begin{aligned} m_1 &= \frac{1}{(a+b)^2}, \\ m_2 &= \frac{1}{(a-b)^2}, \\ m_3 &= \frac{1}{a^2}. \end{aligned} \quad (17)$$

With these eigenvalues we obtain the neutrino mass sum rule

$$\frac{1}{\sqrt{m_1}} = \frac{2}{\sqrt{m_3}} - \frac{1}{\sqrt{m_2}}. \quad (18)$$

which is, indeed, of the type given in Eq. (4).

III. PHENOMENOLOGY

A. Neutrinoless double beta decay

Using the symmetric parametrization of the lepton mixing matrix [10, 11] we can obtain the general expression of the mass parameter $|m_{ee}|$ which determines the $0\nu\beta\beta$ decay amplitude as

$$|m_{ee}| = \left| \sum_j U_{ej}^2 m_j \right| = \begin{cases} |c_{12}^2 c_{13}^2 m_1 e^{2i\alpha} + s_{12}^2 c_{13}^2 m_2 e^{2i\beta} + s_{13}^2 m_3 e^{2i\delta}| & \text{(PDG)}, \\ |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{2i\phi_{12}} + s_{13}^2 m_3 e^{2i\phi_{13}}| & \text{(symmetrical)}. \end{cases} \quad (19)$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, $m_i, i = 1, 2, 3$ the neutrino masses, and we adopt the symmetric parametrization where ϕ_{12} and ϕ_{13} are the two Majorana phases.

By varying the neutrino oscillation parameters in their allowed range one can plot $|m_{ee}|$ in terms of the lightest neutrino mass. Depending on which is the lightest neutrino one can have two different spectra, normal and inverse hierarchy, respectively. In the latter case one has a lower bound, on quite general grounds, as in this case there can be no destructive interference between the light neutrinos.

In the present scheme, however, as noted in Ref. [16], the neutrino mass sum-rule can be interpreted geometrically as a triangle in the complex plane, its area providing a measure of the Majorana CP violation. Then, fixing the (ξ, χ) parameters for each model one can, in principle, determine the two Majorana CP violating phases.

As a result there is a lower bound on $|m_{ee}|$ even in the case of normal hierarchy (for other schemes of this type see, for example, Ref. [16] and references therein), as illustrated in Fig. 3.

B. Quark sector

Quarks are introduced as in table III where, differently from the lepton sector, we assign the first and second families to a doublet representation of S_4 and the third family to a singlet of S_4 , namely $Q_D = (Q_1, Q_2) \sim 2$,

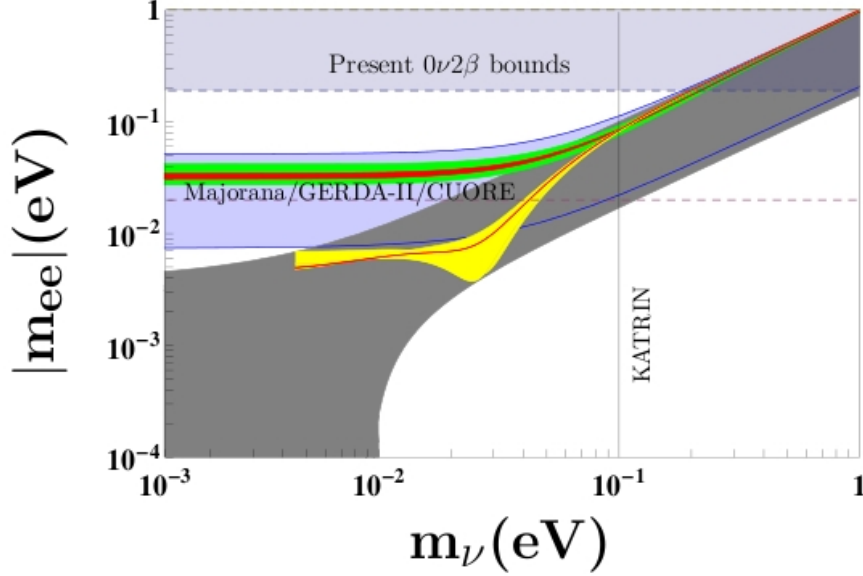


Figure 3: $|m_{ee}|$ as a function of the lightest neutrino mass corresponding to the mass sum-rule in Eq. (4). The grey and dark bands correspond to generic normal and inverse hierarchy regions, while the green and yellow bands correspond to our flavour prediction varying the values of oscillation parameters in their 3σ C.L. range. The thin red bands correspond to the TBM limit. For references to the experiments see [6–9, 28]

$q_{R_D} = (q_{R_1}, q_{R_2}) \sim 2$, $Q_s = Q_3 \sim 1_1$ and $= q_{R_3} \sim 1_1$. We add flavons $\phi_{D,S}$ in doublet and singlet representations of

	\bar{Q}_D	\bar{Q}_S	u_{R_D}	u_{R_S}	d_{R_D}	d_{R_S}	ϕ_D	ϕ_S
$SU(2)$	2	2	1	1	1	1	1	1
S_4	2	1_1	2	1_1	2	1_1	2	1_1
Z_3	ω	ω	ω^2	ω^2	ω	ω	ω	ω
Z_2	+	+	-	-	-	-	-	-

Table III: Quark sector and their transformation properties under the Z_3 , and Z_2 flavor symmetries

the S_4 ,

$$\mathcal{L}_q^d = (Y_1^d \bar{Q}_S d_{R_S} \phi_S + Y_2^d \bar{Q}_D \phi_D d_{R_D} + Y_3^d \bar{Q}_D d_{R_D} \phi_S + Y_4^d \bar{Q}_D \phi_D d_{R_S} + Y_5^d \bar{Q}_S \phi_D d_{R_D}) h / \Lambda + h.c., \quad (20)$$

$$\mathcal{L}_q^u = (Y_1^u \bar{Q}_S u_{R_S} \tilde{\phi}_S + Y_2^u \bar{Q}_D \tilde{\phi}_D u_{R_D} + Y_3^u \bar{Q}_D u_{R_D} \tilde{\phi}_S + Y_4^u \bar{Q}_D \tilde{\phi}_D u_{R_S} + Y_5^u \bar{Q}_S \tilde{\phi}_D u_{R_D}) \tilde{h} / \Lambda + h.c., \quad (21)$$

As in the charged lepton sector the dimension five operators can be given in terms of renormalizable interaction by introducing suitable messenger fields. Taking the VEV of ϕ_D in the direction (we verified that it is a possible solution of the potential)

$$\langle \phi_D \rangle \sim (-\sqrt{3}, 1), \quad (22)$$

the mass matrix for the quarks is

$$\mathbf{M}_{u(d)} = \begin{pmatrix} m_1^{u(d)} + m_2^{u(d)} & -\sqrt{3} m_2^{u(d)} & -\sqrt{3} m_5^{u(d)} \\ -\sqrt{3} m_2^{u(d)} & m_1^{u(d)} - m_2^{u(d)} & m_5^{u(d)} \\ -\sqrt{3} m_4^{u(d)} & m_4^{u(d)} & m_3^{u(d)} \end{pmatrix}, \quad (23)$$

which is very similar to the one proposed in Ref. [29] where a fit of the quarks masses and mixing has been performed and we refer to that paper for more detail.

C. Finite θ_{13} value

As we have discussed so far the model leads to the tribimaximal mixing pattern. However by coupling an extra S_4 -doublet flavon field one can obtain corrections from the charged lepton sector which induce nonzero values of θ_{13} as recently suggested by the T2K [14] and Double-Chooz [15] results [2] including also recent reactor flux calculations.

For example, consider a flavon scalar doublet under S_4 , $\Phi \sim \mathbf{2}$ and transforming as $(\omega, +)$ under $Z_3 \times Z_2$. In the Lagrangian we must then include the term

$$(\bar{L}l_R)h\Phi. \quad (24)$$

This is a dimension five operator which can be obtained from a renormalizable Lagrangian by means of the messenger fields χ, χ^c of table I as shown in figure (2). Assuming that Φ acquires VEV $\langle \Phi \rangle = (u_1, u_2)$, a natural vacuum alignment is $u_1 = -\sqrt{3}u_2$, since this is consistent with the previous alignments in Eq. (12). Using multiplication rules in Appendix A one finds that the contribution from this term to the charged lepton mass matrix is

$$\delta M_l = \begin{pmatrix} -\sqrt{\frac{2}{3}}vu_2 & 0 & 0 \\ 0 & \sqrt{\frac{1}{2}}vu_1 + \sqrt{\frac{1}{6}}vu_2 & 0 \\ 0 & 0 & -\sqrt{\frac{1}{2}}vu_1 + \sqrt{\frac{1}{6}}vu_2 \end{pmatrix} \quad (25)$$

which modifies the diagonal entries δM_l in the charged lepton mass matrix, M_l , so that the total $M_l + \delta M_l$ is no longer diagonalized by U_ω . This way one can induce a potentially “large” value for θ_{13} , as hinted by recent experiments [14, 15], and also potential departures of the solar and atmospheric angles from their TBM values. Moreover, in the presence of a nonzero θ_{13} one finds correlations among these neutrino mixing angles. The most interesting of these involves the solar angle, as illustrated in Fig. 4. The horizontal dot-dashed line indicates the best fit in the global analysis in Ref. [2]. On the other hand the region in the vertical band delimited by the dotted lines corresponds to the 3σ region found in Ref. [2]. One sees how the departure of the solar θ_{12} from its TBM value can be substantial and large θ_{13} values require the solar angle to lie below the TBM prediction¹. A nonzero θ_{13} would also open the way also for the phenomenon of CP violation in neutrino oscillations, one of the central goals of the upcoming generation of long baseline oscillation studies [30, 31].

¹ For technical simplicity we have varied the charged lepton masses well above what is allowed by their current determinations. The real correlation is expressed as a subband of this

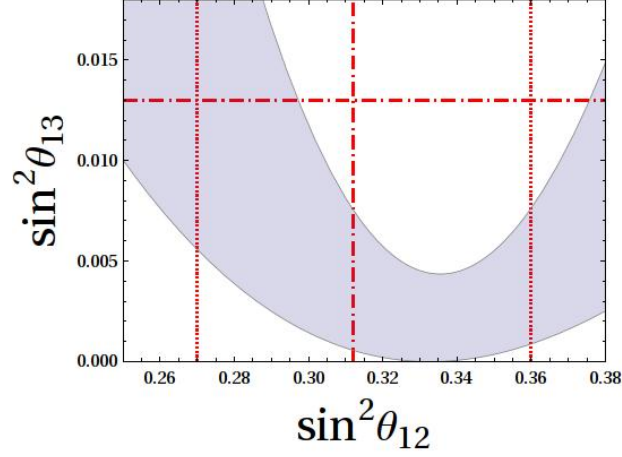


Figure 4: Correlations between reactor and solar neutrino mixing angles. See text for explanations.

Acknowledgments

This work was supported by the Spanish MEC under grants FPA2011-22975 and MULTIDARK CSD2009-00064 (Consolider-Ingenio 2010 Programme), by Prometeo/2009/091 (Generalitat Valenciana), by the EU ITN UNILHC PITN-GA-2009-237920. S. M. is supported by a Juan de la Cierva contract. E. P. is supported by CONACyT (Mexico). A. D. R. is supported by Fundación Carolina and would like to thank hospitality at IFIC while this work was carried out and Alfredo Aranda for his helpful suggestions. L. D. is supported by a CSIC JAE predoctoral fellowship.

Appendix A: S_4 group

The S_4 group is the discrete group given by the four objects permutations. It contains 24 elements and can be obtained from two generators, S and T , satisfying:

$$S^4 = T^3 = 1, \quad ST^2S = T. \quad (\text{A1})$$

The S_4 irreducible representations are two singlets, $\mathbf{1}_1, \mathbf{1}_2$, one doublet, $\mathbf{2}$, and two triplets, $\mathbf{3}_1, \mathbf{3}_2$. The product rules are given by (for more details see [32].)

$$\begin{aligned} \mathbf{1}_i \times \mathbf{1}_j &= \mathbf{1}_{(i+j) \bmod 2 + 1} \quad \forall i, j \\ \mathbf{2} \times \mathbf{1}_i &= \mathbf{2} \quad \forall i \\ \mathbf{3}_i \times \mathbf{1}_j &= \mathbf{1}_{(i+j) \bmod 2 + 1} \quad \forall i, j \\ \mathbf{3}_i \times \mathbf{2} &= \mathbf{3}_1 + \mathbf{3}_2 \quad \forall i \\ \mathbf{3}_1 \times \mathbf{3}_2 &= \mathbf{1}_2 + \mathbf{2} + \mathbf{3}_1 + \mathbf{3}_2 \\ \mathbf{2} \times \mathbf{2} &= \mathbf{1}_1 + \mathbf{2} + \mathbf{1}_2 \\ \mathbf{3}_i \times \mathbf{3}_i &= \mathbf{1}_1 + \mathbf{2} + \mathbf{3}_1 + \mathbf{3}_2 \quad \forall i \end{aligned} \quad (\text{A2})$$

where we can introduce the notation $[\mu \times \mu]$ and $\{\mu \times \mu\}$ for the symmetric and antisymmetric part of $\mu \times \mu$ respectively:

$$\begin{aligned} [\mathbf{2} \times \mathbf{2}] &= \mathbf{1}_1 + \mathbf{2}, \quad \{\mathbf{2} \times \mathbf{2}\} = \mathbf{1}_2 \\ [\mathbf{3}_i \times \mathbf{3}_i] &= \mathbf{1}_1 + \mathbf{2} + \mathbf{3}_1, \quad \{\mathbf{3}_i \times \mathbf{3}_i\} = \mathbf{3}_2 \quad \forall i \end{aligned} \quad (\text{A3})$$

Given the following representations

$$A, A' \sim \mathbf{1}_1, B, B' \sim \mathbf{1}_2, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} a'_1 \\ a'_2 \end{pmatrix} \sim 2, \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \begin{pmatrix} b'_1 \\ b'_2 \\ b'_3 \end{pmatrix} \sim \mathbf{3}_1, \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \begin{pmatrix} c'_1 \\ c'_2 \\ c'_3 \end{pmatrix} \sim \mathbf{3}_2. \quad (\text{A4})$$

As the representation matrices can be choosen all reals, the conjugate representations transform in the same way.

For the product of one dimensional representations the Clebsch Gordan coefficients are the trivial products of representations, and also for the product of the $\mathbf{1}_1$ singlet with any non-trivial representation

$$\begin{pmatrix} Aa_1 \\ Aa_2 \end{pmatrix} \sim 2, \begin{pmatrix} Ab_1 \\ Ab_2 \\ Ab_3 \end{pmatrix} \sim \mathbf{3}_1, \begin{pmatrix} Ac_1 \\ Ac_2 \\ Ac_3 \end{pmatrix} \sim \mathbf{3}_2. \quad (\text{A5})$$

For the product with the $\mathbf{1}_2$ singlet

$$\begin{pmatrix} -Ba_2 \\ Ba_1 \end{pmatrix} \sim 2, \begin{pmatrix} Ab_1 \\ Ab_2 \\ Ab_3 \end{pmatrix} \sim \mathbf{3}_2, \begin{pmatrix} Ac_1 \\ Ac_2 \\ Ac_3 \end{pmatrix} \sim \mathbf{3}_1. \quad (\text{A6})$$

The Clebsch Gordan coefficients for the product $\mathbf{2} \times \mathbf{2}$ are

$$\begin{aligned} a_1 a'_1 + a_2 a'_2 &\sim \mathbf{1}_1 \\ -a_1 a'_2 + a_2 a'_1 &\sim \mathbf{1}_2 \\ \begin{pmatrix} a_1 a'_2 + a_2 a'_1 \\ a_1 a'_1 - a_2 a'_2 \end{pmatrix} &\sim 2, \end{aligned} \quad (\text{A7})$$

for $\mathbf{3}_1 \times \mathbf{3}_1$

$$\begin{aligned} \sum_{j=1}^3 b_j b'_j &\sim \mathbf{1}_1, \\ \begin{pmatrix} \frac{1}{\sqrt{2}}(b_2 b'_2 - b_3 b'_3) \\ \frac{1}{\sqrt{6}}(-2b_1 b'_1 + b_2 b'_2 + b_3 b'_3) \end{pmatrix} &\sim 2, \\ \begin{pmatrix} b_2 b'_3 + b_3 b'_2 \\ b_1 b'_3 + b_3 b'_1 \\ b_1 b'_2 + b_2 b'_1 \end{pmatrix} &\sim \mathbf{3}_1, \quad \begin{pmatrix} b_3 b'_2 - b_2 b'_3 \\ b_1 b'_3 - b_3 b'_1 \\ b_2 b'_1 - b_1 b'_2 \end{pmatrix} &\sim \mathbf{3}_2, \end{aligned} \quad (\text{A8})$$

for $\mathbf{3}_2 \times \mathbf{3}_2$

$$\begin{aligned} \sum_{j=1}^3 c_j c'_j &\sim \mathbf{1}_1, \\ \begin{pmatrix} \frac{1}{\sqrt{2}}(c_2 c'_2 - c_3 c'_3) \\ \frac{1}{\sqrt{6}}(-2c_1 c'_1 + c_2 c'_2 + c_3 c'_3) \end{pmatrix} &\sim 2, \\ \begin{pmatrix} c_2 c'_3 + c_3 c'_2 \\ c_1 c'_3 + c_3 c'_1 \\ c_1 c'_2 + c_2 c'_1 \end{pmatrix} &\sim \mathbf{3}_1, \quad \begin{pmatrix} c_3 c'_2 - c_2 c'_3 \\ c_1 c'_3 - c_3 c'_1 \\ c_2 c'_1 - c_1 c'_2 \end{pmatrix} &\sim \mathbf{3}_2. \end{aligned} \quad (\text{A9})$$

For the couplings $\mathbf{2} \times \mathbf{3}_1$ and $\mathbf{2} \times \mathbf{3}_2$, we have respectively

$$\begin{aligned} \begin{pmatrix} a_2 b_1 \\ -\frac{1}{2}(\sqrt{3}a_1 b_2 + a_2 b_2) \\ \frac{1}{2}(\sqrt{3}a_1 b_3 - a_2 b_3) \end{pmatrix} &\sim \mathbf{3}_1, & \begin{pmatrix} a_1 c_1 \\ \frac{1}{2}(\sqrt{3}a_2 c_2 - a_1 c_2) \\ -\frac{1}{2}(\sqrt{3}a_2 c_3 + a_1 c_3) \end{pmatrix} &\sim \mathbf{3}_1, \\ \begin{pmatrix} a_1 b_1 \\ \frac{1}{2}(\sqrt{3}a_2 b_2 - a_1 b_2) \\ -\frac{1}{2}(\sqrt{3}a_2 b_3 + a_1 b_3) \end{pmatrix} &\sim \mathbf{3}_2, & \begin{pmatrix} a_2 c_1 \\ -\frac{1}{2}(\sqrt{3}a_1 c_2 + a_2 c_2) \\ \frac{1}{2}(\sqrt{3}a_1 c_3 - a_2 c_3) \end{pmatrix} &\sim \mathbf{3}_2. \end{aligned} \quad (\text{A10})$$

And finally, for the $\mathbf{3}_1 \times \mathbf{3}_2$ product

$$\begin{aligned} \sum_{j=1}^3 b_j c_j &\sim \mathbf{1}_2, \\ \begin{pmatrix} \frac{1}{\sqrt{6}}(2b_1 c_1 - b_2 c_2 - b_3 c_3) \\ \frac{1}{\sqrt{2}}(b_2 c_2 - b_3 c_3) \end{pmatrix} &\sim \mathbf{2}, \\ \begin{pmatrix} b_3 c_2 - b_2 c_3 \\ b_1 c_3 - b_3 c_1 \\ b_2 c_1 - b_1 c_2 \end{pmatrix} &\sim \mathbf{3}_1, & \begin{pmatrix} b_2 c_3 + b_3 c_2 \\ b_1 c_3 + b_3 c_1 \\ b_1 c_2 + b_2 c_1 \end{pmatrix} &\sim \mathbf{3}_2. \end{aligned} \quad (\text{A11})$$

Appendix B: Yukawa couplings

Each term in the Lagrangian for neutrinos and charged leptons can be decomposed by components as:

$$\mu S \cdot S \sigma = \mu(S_1 S_1 + S_2 S_2 + S_3 S_3) \sigma, \quad (\text{B1})$$

$$Y_D \bar{L} \cdot \nu_R h = Y_D (\bar{L}_1 \nu_{1R} + \bar{L}_2 \nu_{2R} + \bar{L}_3 \nu_{3R}) h, \quad (\text{B2})$$

$$Y_\nu (\nu_R \cdot S) \phi_\nu = Y_\nu [(\nu_{2R} S_{3R} + \nu_{3R} S_2) \phi_{\nu 1} + (\nu_{1R} S_{3R} + \nu_{3R} S_1) \phi_{\nu 2} + (\nu_{1R} S_{2R} + \nu_{2R} S_1) \phi_{\nu 3}], \quad (\text{B3})$$

$$Y_{\nu'} (\nu_R \cdot S) \phi_{\nu'} = Y_{\nu'} (\nu_{1R} S_1 + \nu_{2R} S_2 + \nu_{3R} S_3) \phi_{\nu'}, \quad (\text{B4})$$

$$\frac{y_l}{\Lambda} (\bar{L} l_R) h \phi_l = \frac{y_l}{\Lambda} [(\bar{L}_2 l_{3R} + \bar{L}_3 l_{2R}) h \phi_{l1} + (\bar{L}_1 l_{3R} + \bar{L}_3 l_{1R}) h \phi_{l2} + (\bar{L}_1 l_{2R} + \bar{L}_2 l_{1R}) h \phi_{l3}] \quad (\text{B5})$$

$$\frac{y'_l}{\Lambda} (\bar{L} l_R) h \phi_{l'} = \frac{y'_l}{\Lambda} [(\bar{L}_3 l_{2R} - \bar{L}_2 l_{3R}) h \phi_{l'1} + (\bar{L}_1 l_{3R} - \bar{L}_3 l_{1R}) h \phi_{l'2} + (\bar{L}_2 l_{1R} - \bar{L}_1 l_{2R}) h \phi_{l'3}] \quad (\text{B6})$$

$$\frac{y''_l}{\Lambda} (\bar{L} l_R) h \phi_{l''} = \frac{y''_l}{\Lambda} (\bar{L}_1 l_{1R} + \bar{L}_2 l_{2R} + \bar{L}_3 l_{3R}) h \phi_{l''} \quad (\text{B7})$$

Appendix C: Scalar potential

The most general renormalizable scalar potential is (without write the S_4 products explicitly):

$$V = V(h) + V(\sigma) + V(\Phi_\nu) + V(\Phi_{\nu'}) + V(\Phi_l) + V(\Phi_{l'}) + V(\Phi_{l''}) \quad (C1)$$

$$+ V(\Phi_\nu, \Phi_{\nu'}, \Phi_l, \Phi_{l'}, \Phi_{l''}) + V(\sigma, h, \Phi_\nu, \Phi_{\nu'}, \Phi_l, \Phi_{l'}, \Phi_{l''}), \quad (C2)$$

with

$$V(h) = \mu_h h^\dagger h + \lambda_h (h^\dagger h)(h^\dagger h),$$

$$V(\sigma) = \mu_\sigma \sigma^\dagger \sigma + \lambda_\sigma (\sigma^\dagger \sigma)(\sigma^\dagger \sigma)$$

$$V(\Phi_\nu) = \mu_1 (\Phi_\nu^\dagger \Phi_\nu) + \sum_i \lambda_j^\nu \{ \Phi_\nu^\dagger \Phi_\nu \Phi_\nu^\dagger \Phi_\nu \}_i$$

$$V(\Phi_{\nu'}) = \mu_2 (\Phi_{\nu'}^\dagger \Phi_{\nu'}) + \sum_i \lambda_j^{\nu'} \{ \Phi_{\nu'}^\dagger \Phi_{\nu'} \Phi_{\nu'}^\dagger \Phi_{\nu'} \}_i$$

$$V(\Phi_l) = \mu_3 (\Phi_l^\dagger \Phi_l) + \sum_i \lambda_i^l \{ \Phi_l^\dagger \Phi_l \Phi_l^\dagger \Phi_l \}_i + \sum_i \kappa_i \{ (\Phi_l \Phi_l) \Phi_l + h.c. \}_i,$$

$$V(\Phi_{l'}) = \mu_4 (\Phi_{l'}^\dagger \Phi_{l'}) + \sum_i \lambda_i^{l'} \{ \Phi_{l'}^\dagger \Phi_{l'} \Phi_{l'}^\dagger \Phi_{l'} \}_i + \sum_i \kappa_i \{ (\Phi_{l'} \Phi_{l'}) \Phi_{l'} + h.c. \}_i,$$

$$V(\Phi_{l''}) = \mu_5 (\Phi_{l''}^\dagger \Phi_{l''}) + \sum_i \lambda_i^{l''} \{ \Phi_{l''}^\dagger \Phi_{l''} \Phi_{l''}^\dagger \Phi_{l''} \}_i + h.c.,$$

$$\begin{aligned} V(\sigma, h, \Phi_\nu, \Phi_{\nu'}, \Phi_l, \Phi_{l'}, \Phi_{l''}) = & \lambda^{h\sigma} (h^\dagger h)(\sigma^\dagger \sigma) + \lambda^{\nu\sigma} (\Phi_\nu^\dagger \Phi_\nu)(\sigma^\dagger \sigma) + \lambda^{\nu'\sigma} (\Phi_{\nu'}^\dagger \Phi_{\nu'})(\sigma^\dagger \sigma) \\ & + \lambda^{l\sigma} (\Phi_l^\dagger \Phi_l)(\sigma^\dagger \sigma) + \lambda^{l'\sigma} (\Phi_{l'}^\dagger \Phi_{l'})(\sigma^\dagger \sigma) + \lambda^{l''\sigma} (\Phi_{l''}^\dagger \Phi_{l''})(\sigma^\dagger \sigma), \end{aligned}$$

$$\begin{aligned}
V(\Phi_\nu, \Phi_{\nu'}, \Phi_l, \Phi_{l'}, \Phi_{l''}) = & \sum_i \kappa_i \{ \Phi_l \Phi_l \Phi_{l'} \}_i + \sum_i \kappa_i \{ \Phi_l \Phi_{l'} \Phi_{l'} \}_i \\
& + \sum_i \kappa_i \{ \Phi_l \Phi_l \Phi_{l''} \}_i + \sum_i \kappa_i \{ \Phi_{l'} \Phi_{l'} \Phi_{l''} \}_i \\
& + \sum_i \kappa_i \{ \Phi_l^\dagger \Phi_\nu \Phi_\nu \}_i + \sum_i \kappa_i \{ \Phi_{l'}^\dagger \Phi_\nu \Phi_\nu \}_i + \sum_i \kappa_i \{ \Phi_{l''}^\dagger \Phi_\nu \Phi_\nu \}_i \\
& + \sum_i \kappa_i \{ \Phi_l^\dagger \Phi_\nu \Phi_{\nu'} \}_i + \sum_i \kappa_i \{ \Phi_{l'}^\dagger \Phi_{\nu'} \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{ll'l'} \{ \Phi_l^\dagger \Phi_l^\dagger \Phi_{l'} \Phi_{l'} \}_i + \sum_i \lambda_i^{ll'l'l'} \{ \Phi_l \Phi_l^\dagger \Phi_{l'} \Phi_{l'} \}_i \\
& + \sum_i \lambda_i^{llll'} \{ \Phi_l^\dagger \Phi_l^\dagger \Phi_l \Phi_{l'} \}_i + \sum_i \lambda_i^{llll'} \{ \Phi_{l'}^\dagger \Phi_l^\dagger \Phi_l \Phi_{l'} \}_i \\
& + \sum_i \lambda_i^{ll'l'l'} \{ \Phi_l^\dagger \Phi_{l'}^\dagger \Phi_{l'} \Phi_{l'} \}_i + \sum_i \lambda_i^{llll'} \{ \Phi_l^\dagger \Phi_l^\dagger \Phi_l \Phi_{l''} \}_i \\
& + \sum_i \lambda_i^{llll''} \{ \Phi_l^\dagger \Phi_l^\dagger \Phi_{l'} \Phi_{l''} \}_i + \sum_i \lambda_i^{ll'l'l''} \{ \Phi_l^\dagger \Phi_{l'}^\dagger \Phi_l \Phi_{l''} \}_i \\
& + \sum_i \lambda_i^{ll'l'l''} \{ \Phi_l^\dagger \Phi_{l'}^\dagger \Phi_{l'} \Phi_{l''} \}_i + \sum_i \lambda_i^{ll'l'l''} \{ \Phi_{l'}^\dagger \Phi_{l'}^\dagger \Phi_l \Phi_{l''} \}_i \\
& + \sum_i \lambda_i^{ll'l'l'l''} \{ \Phi_{l'}^\dagger \Phi_{l'}^\dagger \Phi_{l'} \Phi_{l''} \}_i + \sum_i \lambda_i^{ll'l'l''} \{ \Phi_l^\dagger \Phi_l^\dagger \Phi_{l''} \Phi_{l''} \}_i \\
& + \sum_i \lambda_i^{ll'l'l'l''} \{ \Phi_{l'}^\dagger \Phi_{l''}^\dagger \Phi_{l'} \Phi_{l''} \}_i \\
& + \sum_i \lambda_i^{ll\nu\nu} \{ \Phi_l \Phi_l \Phi_\nu \Phi_\nu \}_i + \sum_i \lambda_i^{ll'\nu\nu} \{ \Phi_l \Phi_{l'} \Phi_\nu \Phi_\nu \}_i \\
& + \sum_i \lambda_i^{ll'l'\nu\nu} \{ \Phi_{l'} \Phi_{l'} \Phi_\nu \Phi_\nu \}_i + \sum_i \lambda_i^{ll''\nu\nu} \{ \Phi_l \Phi_{l''} \Phi_\nu \Phi_\nu \}_i \\
& + \sum_i \lambda_i^{ll'l''\nu\nu} \{ \Phi_{l'} \Phi_{l''} \Phi_\nu \Phi_\nu \}_i + \sum_i \lambda_i^{ll''\nu\nu} \{ \Phi_{l''} \Phi_{l''} \Phi_\nu \Phi_\nu \}_i \\
& + \sum_i \lambda_i^{ll\nu\nu} \{ \Phi_l^\dagger \Phi_l \Phi_\nu^\dagger \Phi_\nu \}_i + \sum_i \lambda_i^{ll'\nu\nu} \{ \Phi_l^\dagger \Phi_{l'} \Phi_\nu^\dagger \Phi_\nu \}_i \\
& + \sum_i \lambda_i^{ll'l'\nu\nu} \{ \Phi_{l'}^\dagger \Phi_{l'} \Phi_\nu^\dagger \Phi_\nu \}_i + \sum_i \lambda_i^{ll''\nu\nu} \{ \Phi_l^\dagger \Phi_{l''} \Phi_\nu^\dagger \Phi_\nu \}_i \\
& + \sum_i \lambda_i^{ll'l''\nu\nu} \{ \Phi_{l'}^\dagger \Phi_{l''} \Phi_\nu^\dagger \Phi_\nu \}_i + \sum_i \lambda_i^{ll\nu\nu'} \{ \Phi_l \Phi_l \Phi_\nu \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{ll'\nu\nu'} \{ \Phi_l \Phi_{l'} \Phi_\nu \Phi_{\nu'} \}_i + \sum_i \lambda_i^{ll''\nu\nu'} \{ \Phi_l \Phi_{l''} \Phi_\nu \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{ll'l'\nu\nu'} \{ \Phi_{l'} \Phi_{l'} \Phi_\nu \Phi_{\nu'} \}_i + \sum_i \lambda_i^{ll'l\nu\nu'} \{ \Phi_l^\dagger \Phi_{l'} \Phi_\nu^\dagger \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{ll'l''\nu\nu'} \{ \Phi_{l'}^\dagger \Phi_{l''} \Phi_\nu^\dagger \Phi_{\nu'} \}_i + \sum_i \lambda_i^{ll''\nu\nu'} \{ \Phi_l^\dagger \Phi_{l''} \Phi_\nu^\dagger \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{\nu\nu\nu\nu'} \{ \Phi_\nu^\dagger \Phi_\nu \Phi_\nu^\dagger \Phi_{\nu'} \}_i + \sum_i \lambda_i^{\nu\nu'\nu\nu'} \{ \Phi_\nu^\dagger \Phi_{\nu'} \Phi_\nu^\dagger \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{\nu\nu\nu'\nu'} \{ \Phi_\nu^\dagger \Phi_\nu \Phi_{\nu'}^\dagger \Phi_{\nu'} \}_i
\end{aligned}$$

$$\begin{aligned}
& + \sum_i \lambda_i^{ll'\nu'\nu'} \{ \Phi_l \Phi_{l'} \Phi_{\nu'} \Phi_{\nu'} \}_i + \sum_i \lambda_i^{l'l'\nu'\nu'} \{ \Phi_{l'} \Phi_{l'} \Phi_{\nu'} \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{l''l''\nu'\nu'} \{ \Phi_{l''} \Phi_{l''} \Phi_{\nu'} \Phi_{\nu'} \}_i + \sum_i \lambda_i^{l''\nu'\nu'} \{ \Phi_l^\dagger \Phi_{l''} \Phi_{\nu'}^\dagger \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{ll'\nu'\nu'} \{ \Phi_l^\dagger \Phi_l \Phi_{\nu'}^\dagger \Phi_{\nu'} \}_i + \sum_i \lambda_i^{l'l'\nu'\nu'} \{ \Phi_{l'}^\dagger \Phi_{l'} \Phi_{\nu'}^\dagger \Phi_{\nu'} \}_i \\
& + \sum_i \lambda_i^{l''\nu'\nu'} \{ \Phi_{l''}^\dagger \Phi_{l''} \Phi_{\nu'}^\dagger \Phi_{\nu'} \}_i + \sum_i \lambda_i^{l''l''\nu'\nu'} \{ \Phi_{l''}^\dagger \Phi_{l''} \Phi_{\nu'}^\dagger \Phi_{\nu'} \}_i \\
& + h.c.,
\end{aligned}$$

where $\sum_i \lambda_i \{ \}_i$, $\sum_i \kappa_i \{ \}_i$ sums over all possible ways to group the fields inside the brackets and make the product of representations in order to obtain a singlet.

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